Benha University

Faculty Of Engineering at Shoubra

# ECE 122

# Electrical Circuits (2)(2017/2018)

# Lecture (6)

# Magnetically Coupled Circuits

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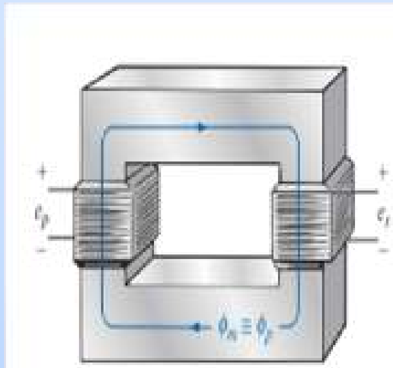
# Magnetically Coupled Circuits

- The circuits we have considered previously may be regarded as conductively coupled, because one loop affects the neighboring loop through current conduction.
- When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be magnetically coupled.

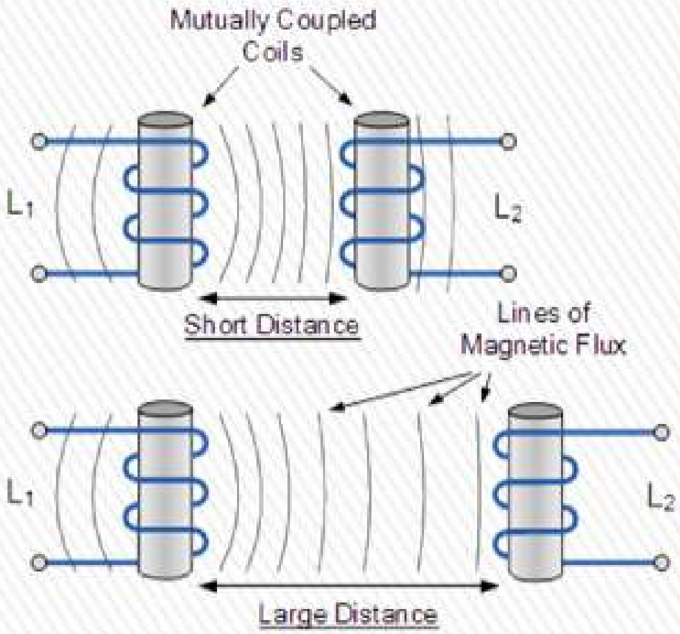
- Example: transformer
  - An electrical device designed on the basis of the concept of magnetic coupling.
  - Used magnetically coupled coils to transfer energy from one circuit to another.



# Magnetically Coupled Circuits

- **Mutual Inductance** is the basic operating principal of many application such as transformer, magnetic levitation trains and other electrical component that interacts with another magnetic field.

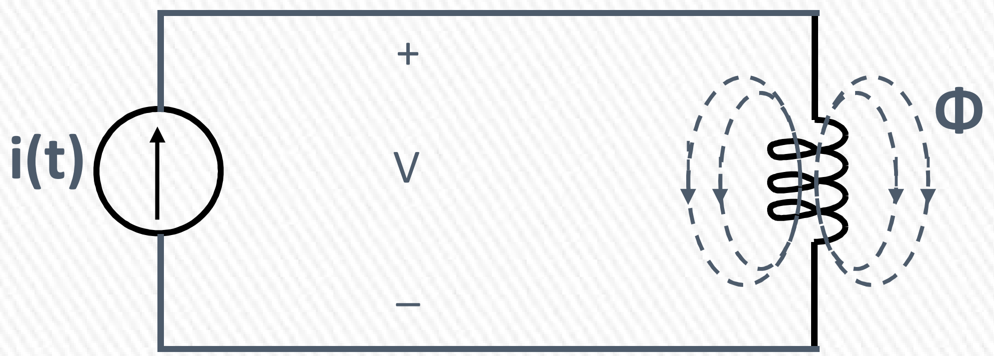
- But mutual inductance can also be a bad thing as “stray” or “leakage” inductance from a coil can interfere with the operation of another adjacent component by means of electromagnetic induction, so some form of protection may be needed



# a) Self Inductance

» It called *self inductance* because it relates the voltage induced in a coil by a time varying current in the same coil.

» Consider a single inductor with N number of turns when current, *i* flows through the coil, a magnetic flux, Φ is produces around it.



# Self Inductance

- According to Faraday's Law, the voltage, *v* induced in the coil is proportional to N number of turns and rate of change of the magnetic flux, Φ;

$$v = N\frac{d\phi}{dt} .......(1)$$

- But a change in the flux $\Phi$ is caused by a change in current, *i*.

Hence;

$$\frac{d\phi}{dt} = \frac{d\phi}{di}\frac{di}{dt} .......(2)$$

# a) Self Inductance

Thus, (2) into (1) yields;

$$v = N\frac{d\phi}{di}\frac{di}{dt} \text{.......(3)}$$

or

$$v = L\frac{di}{dt} \text{.......(4)}$$

From equation (3) and (4) the self inductance L is define as;

$$L = N\frac{d\phi}{di} \quad [\mathrm{H}] \text{........( 5)}$$

The unit is in Henrys (H)

# Self Inductance

$$v = L\frac{di}{dt} = N\frac{d\phi}{dt} \qquad \rightarrow L = N\frac{d\phi}{di}$$

For Sinusoidal current: d/dt = jw

V = L d(i)/dt = L (jw) i = j i $X_L$

For Linear System (coils with air-core not iron-core):

L = N ϕ/ i

- Self-Inductance parameters

$$L = \frac{N^2 \mu A}{l} \quad \text{(henries, H)}$$

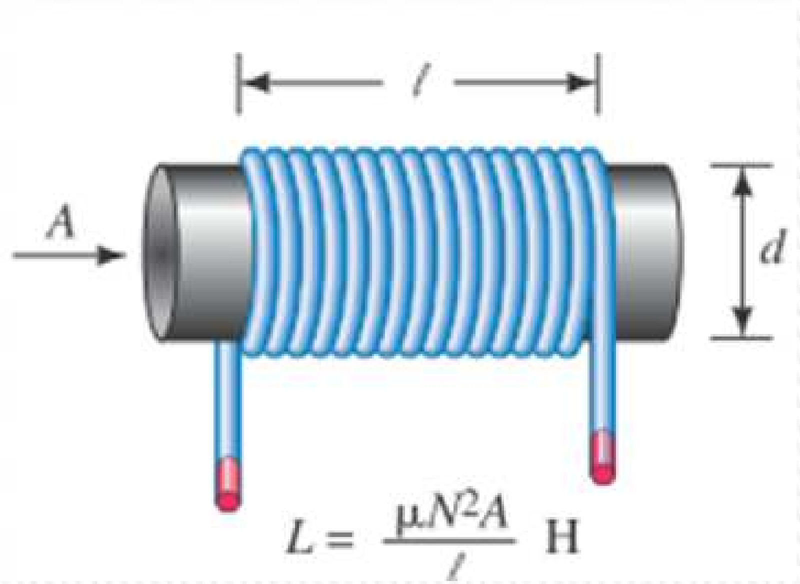
$\mu_o$ is the permeability of free space ($4.\pi.10^{-7}$)

$\mu_r$ is the relative permeability of the soft iron core

N is in the number of coil turns

A is in the cross-sectional area in $m^2$

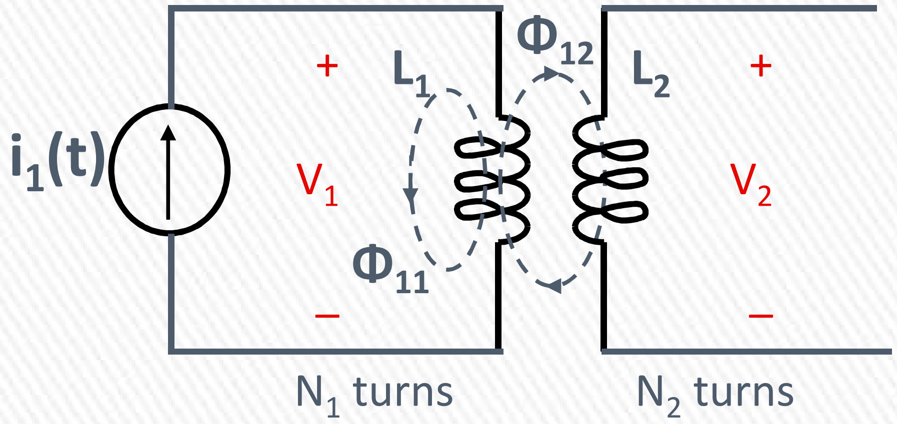
l is the coils length in meters



# Mutual Inductance

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, producing induced voltage.

➢ For simplicity, assume that the second inductor carries no current.



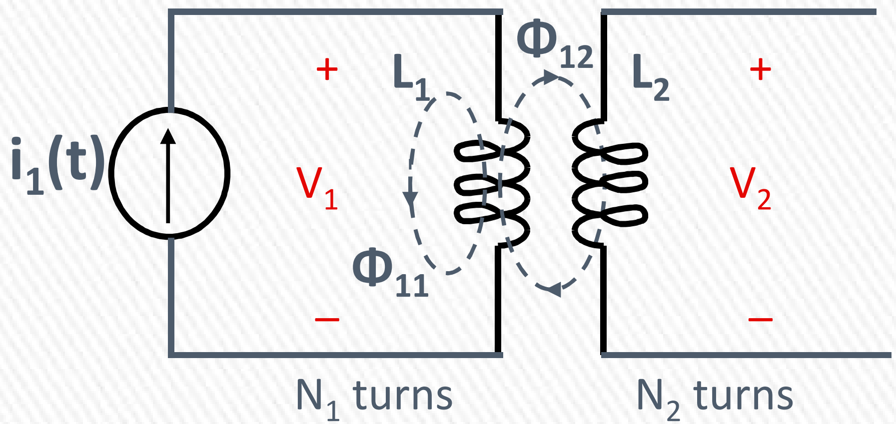

**two coil with self – inductance $L_1$ and $L_2$ which are in close proximity which each other. Coil 1 has $N_1$ turns, while coil 2 has $N_2$ turns.**

# Mutual Inductance

» Magnetic flux $\Phi_1$ from coil 1 has two components;

* $\Phi_{11}$ links only coil 1.

* $\Phi_{12}$ links both coils.




Hence; $\Phi_1 = \Phi_{11} + \Phi_{12}$

Leakage Flux + Linkage Flux

Voltage induces in coil 1

$$v_1 = N_1 \frac{d\phi_{11}}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

Voltage induces in coil 2

$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt} \quad .......(8)$$

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

**Subscript 21 in $M_{21}$ means the mutual inductance on coil 2 due to coil 1**

» Case 2:

Same circuit but let current $i_2$ flow in coil 2.

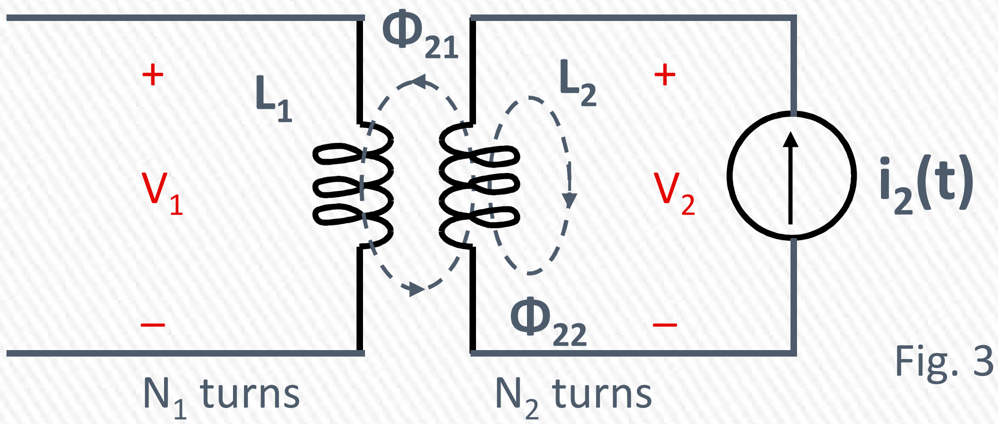



Fig. 3

» The magnetic flux $\Phi_2$ from coil 2 has two components:

* $\Phi_{22}$ links only coil 2.
* $\Phi_{21}$ links both coils.

Hence; $\Phi_2 = \Phi_{21} + \Phi_{22}$ ……. (9)

Thus;

Voltage induced in coil 2

$$v_2 = N_2 \frac{d\phi_{22}}{di_2} \frac{di_2}{dt} = L_2 \frac{di_2}{dt} \text{.......}(10)$$

Voltage induced in coil 1

$$v_1 = N_1 \frac{d\phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt} \text{.......}(11)$$

Subscript 12 in $M_{12}$ means the Mutual Inductance on coil 1 due to coil 2

» Since the two circuits and two current are the same:

$$M_{21} = M_{12} = M$$

» Mutual inductance M is measured in Henrys (H)

## Coupling Coefficient

**Is the fraction of the total flux that links to both coils**

$$k \equiv \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

$$M^2 = \left(N_2 \frac{d\phi_{12}}{di_1}\right)\left(N_1 \frac{d\phi_{21}}{di_2}\right) = \left(N_2 \frac{d(k\phi_1)}{di_1}\right)\left(N_1 \frac{d(k\phi_2)}{di_2}\right) =$$

$$= k^2\left(N_1 \frac{d\phi_1}{di_1}\right)\left(N_2 \frac{d\phi_2}{di_2}\right) = k^2 L_1 L_2$$

$$M = k\sqrt{L_1 L_2}$$

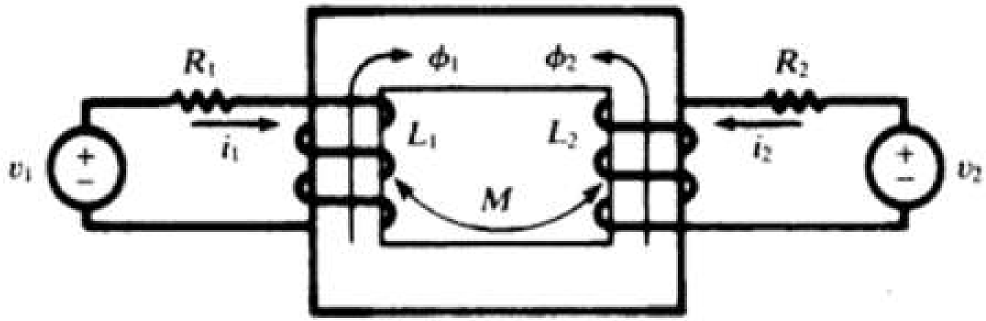
If all of the flux links the coils without any leakage flux, then $k = 1$.

**k depends on the closeness of two coils, their core and their winding.**

# Right-Hand Rule

## ➢ (1) Right-Hand Rule

- **The two coils are on a common core which channels the magnetic flux**




- **To determine the proper signs on the voltages of mutual inductance, apply the right-hand rule to each coil:**

If the fingers wrap around in the direction of the assumed current, the thumb points in the direction of the flux.

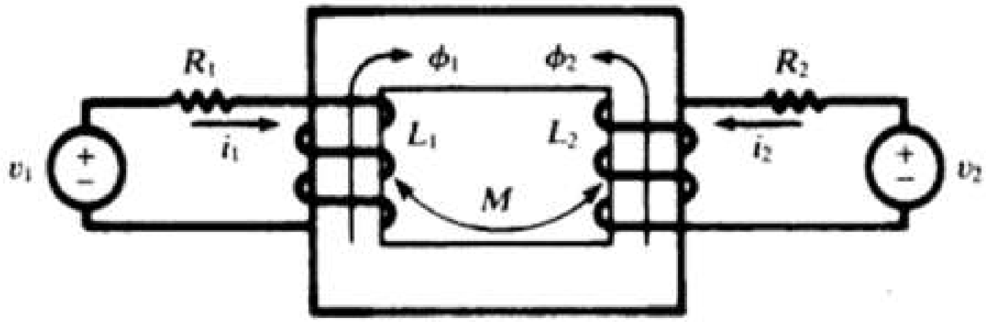
1. If fluxes $\phi_1$ and $\phi_2$ aid one another, then the signs on the voltages of mutual inductance are the same as the signs on the voltages of self-inductance
2. If they oppose each other; a minus sign is used

$$R_1 i_1 + L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} = v_1$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt} = v_2$$

## Polarities in Close Coupling ➢

- **So in our case :**




$$R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = v_1$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = v_2$$

Assuming sinusoidal voltage sources,

$$(R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M \mathbf{I}_2 = \mathbf{V}_1$$

$$-j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2 = \mathbf{V}_2$$

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

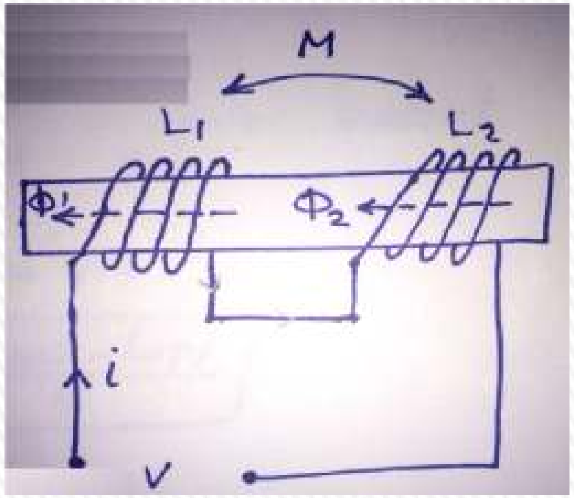
## Series-Aiding and Series opposing Coils

### 1. Series Aiding Coils

$$\mathbf{V} = j\omega L_1 \mathbf{I} + j\omega M \mathbf{I} + j\omega L_2 \mathbf{I} + j\omega M \mathbf{I}$$

$$= j\omega L_{eq} \mathbf{I}$$

where $L_{eq} = L_1 + L_2 + 2M$.

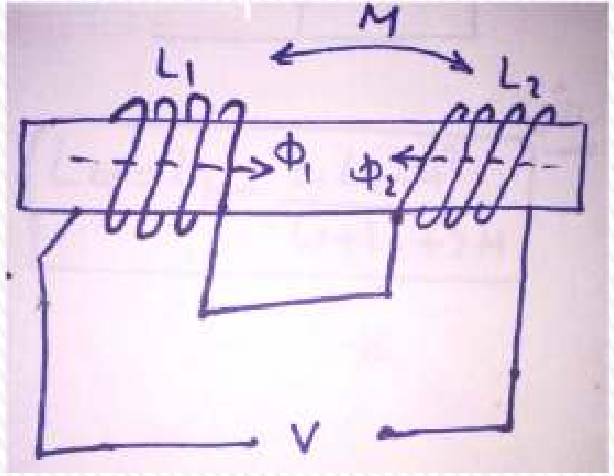



### 2. Series opposing Coils

$$\mathbf{V} = j\omega L_1 \mathbf{I} - j\omega M \mathbf{I} + j\omega L_2 \mathbf{I} - j\omega M \mathbf{I}$$

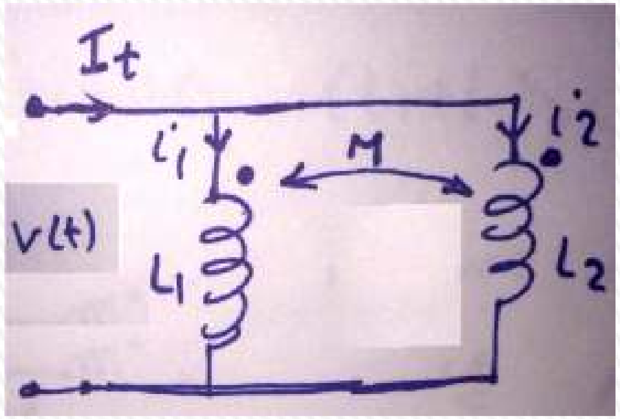
$$= j\omega L_{eq} \mathbf{I}$$

where $L_{eq} = L_1 + L_2 - 2M$.



## Parallel-Aiding and Parallel-opposing Coils

### 1. Parallel Aiding Coils




$$\mathbf{V} = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V} = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

Solving these equations for $\mathbf{I}_1$ and $\mathbf{I}_2$ yields

$$\mathbf{I}_1 = \frac{\mathbf{V}(L_2 - M)}{j\omega(L_1 L_2 - M^2)}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}(L_1 - M)}{j\omega(L_1 L_2 - M^2)}$$

Using KCL gives us

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \frac{\mathbf{V}(L_1 + L_2 - 2M)}{j\omega(L_1 L_2 - M^2)} = \frac{\mathbf{V}}{j\omega L_{eq}}$$

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

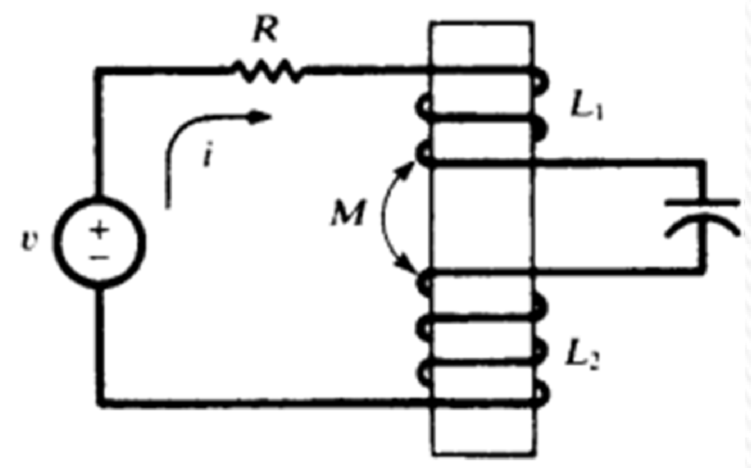
### 2. Parallel opposing Coils

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

# Examples (Schaum's)

$$L' \equiv L_1 + L_2 - 2M.$$

**Series opposing Coils**



# dot convention

# dot convention

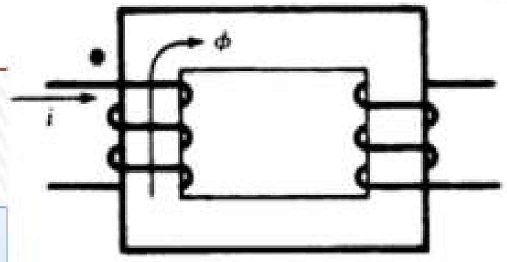
- ✓ Required to determine polarity of “mutual” induced voltage.
- ✓ A dot is placed in the circuit at one end of each of the two magnetically coupled to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil

✓ Steps to assign the dots:

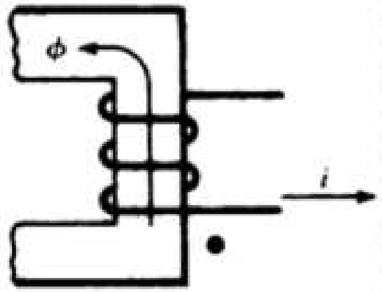
a. select a current direction in one coil and place a dot at the terminal where this current enters the winding.
b. Determine the corresponding flux by application of the right-hand rule

c. The flux of the other winding, according to Lenz’s law, opposes the first flux.
d. Use the right-hand rule to find the natural current direction corresponding to this second flux

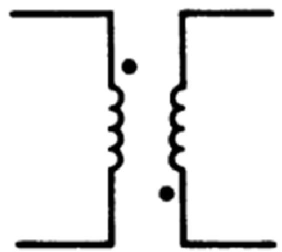
e. Now place a dot at the terminal of the second winding where the natural current leaves the winding.



(a)

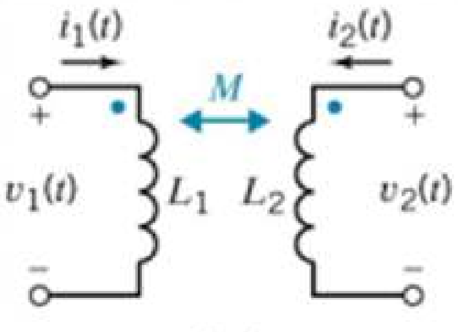


(b)


(c)

# The Dot Rule

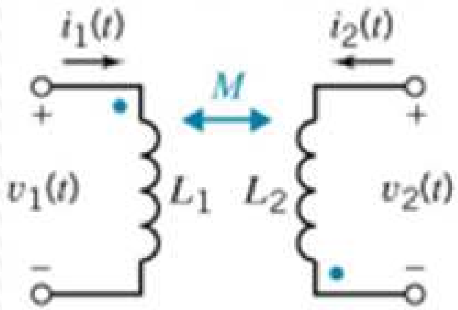
1. When the assumed currents both enter or both leave a pair of coupled coils by the dotted terminals, the signs on the M-terms will be the same as the signs on the L-terms
2. If one current enters by a dotted terminal while the other leaves by a dotted terminal, the signs on the M-terms will be opposite to the signs on the L-terms.



(a)

$$v_1(t) = L_1 \frac{d}{dt} i_1(t) + M \frac{d}{dt} i_2(t)$$

$$v_2(t) = L_2 \frac{d}{dt} i_2(t) + M \frac{d}{dt} i_1(t)$$

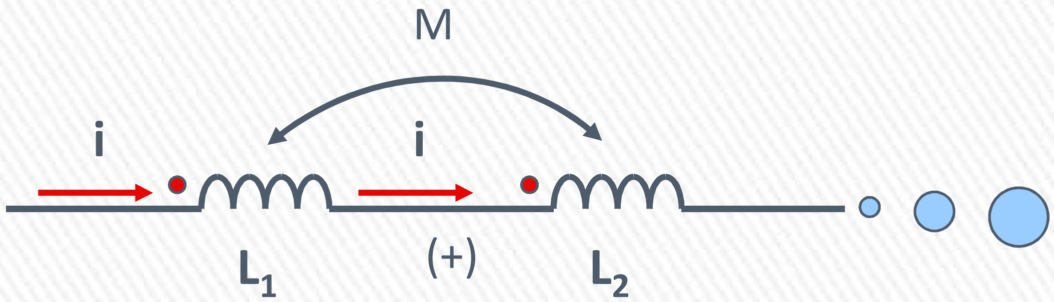


(b)

$$v_1(t) = L_1 \frac{d}{dt} i_1(t) - M \frac{d}{dt} i_2(t)$$

$$v_2(t) = L_2 \frac{d}{dt} i_2(t) - M \frac{d}{dt} i_1(t)$$

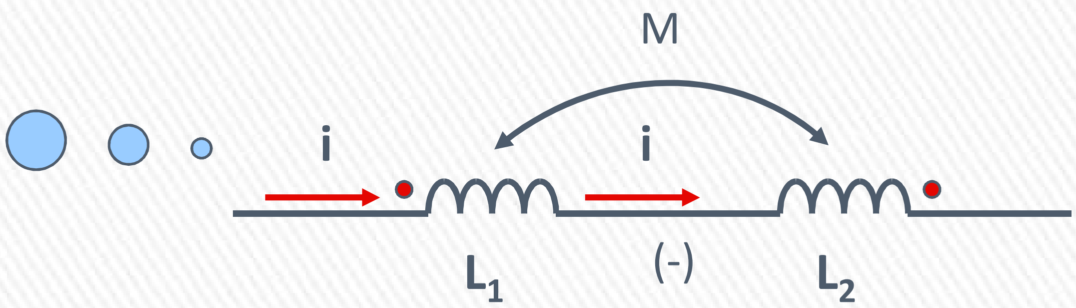
# Dot convention for coils in series




Series – aiding connection

$$L = L_1 + L_2 + 2M$$

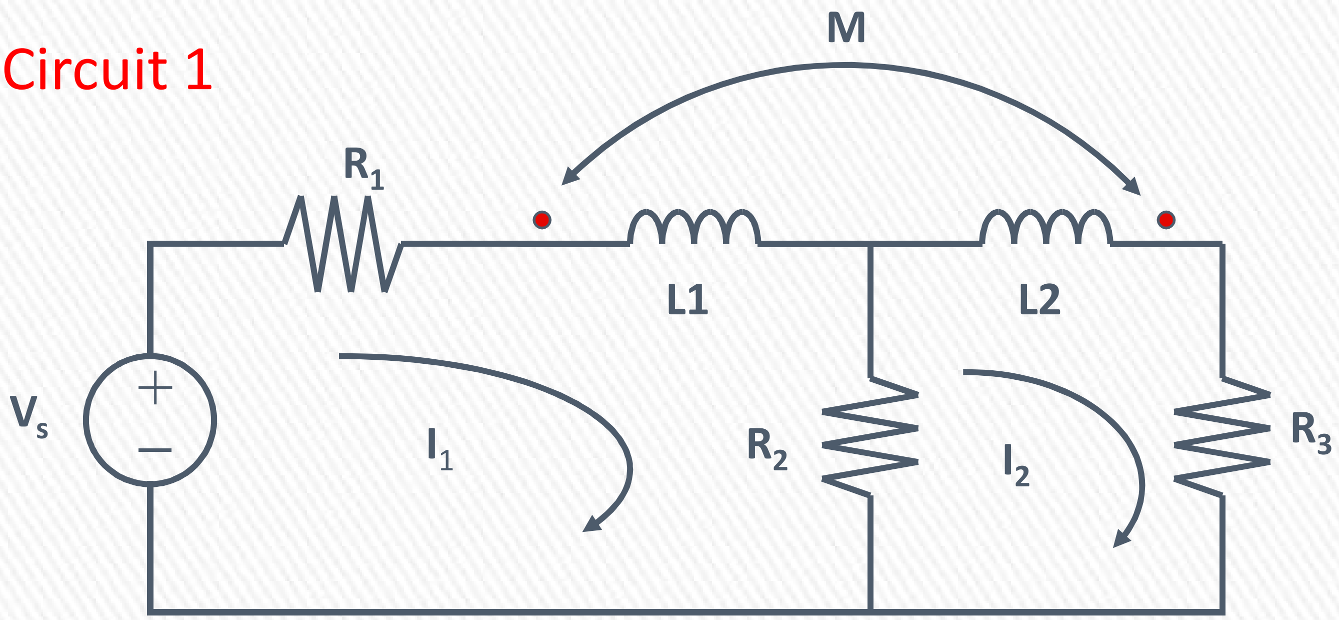
Series – opposing connection




$$L = L_1 + L_2 - 2M$$

Below are examples of the sets of equations derived from basic configurations involving mutual inductance

» Circuit 1

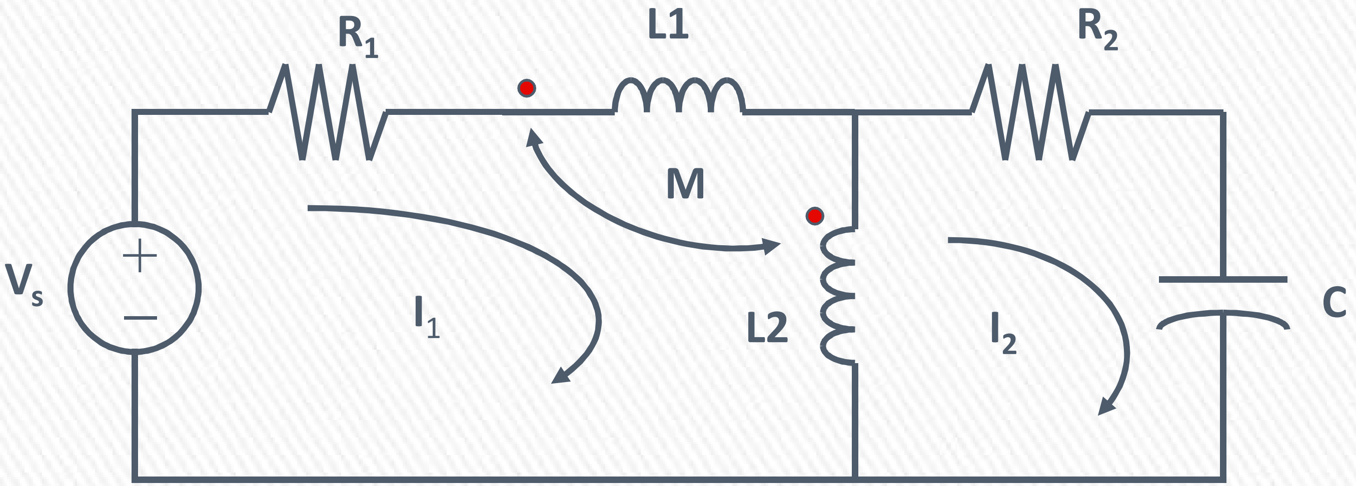



Solution:

$$\text{KVL } \mathrm{I}_1 : (R_1 + R_2 + jwL_1)I_1 - jwMI_2 - R_2 I_2 = V_s .......(1)$$

$$\text{KVL } \mathrm{I}_2 : -R_2 I_1 + (R_2 + R_3 + jwL_2)I_2 - jwMI_1 = 0.......(2)$$

» Circuit 2

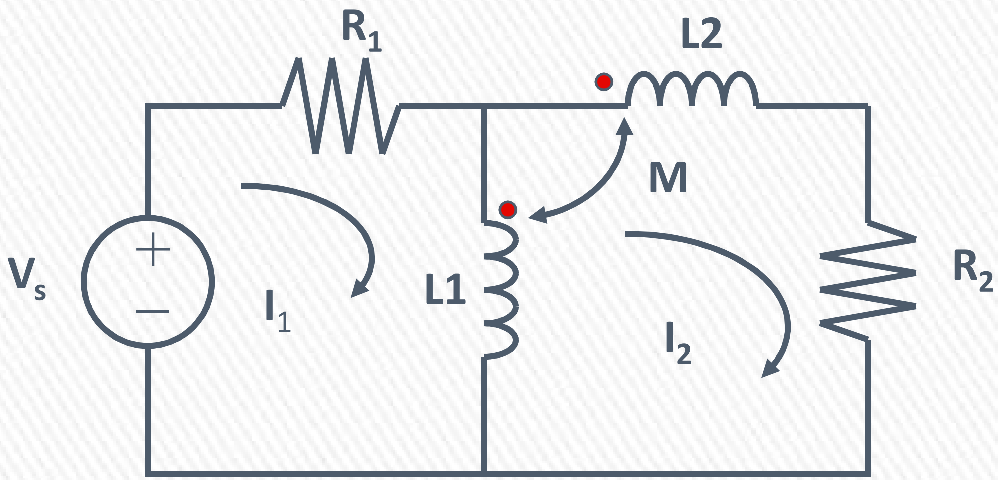



Solution:

$$\text{KVL I}_1 : (R_1 + j\omega L_1 + j\omega L_2)I_1 - j\omega L_2 I_2 + j\omega M(I_1 - I_2) + j\omega M I_1 = V_s \text{.......}(1)$$

$$\text{KVL I}_2 : -j\omega L_2 I_1 + (R_2 + j\omega L_2 - j/\omega c)I_2 - j\omega M I_1 = 0 \text{.......}(2)$$

» Circuit 3

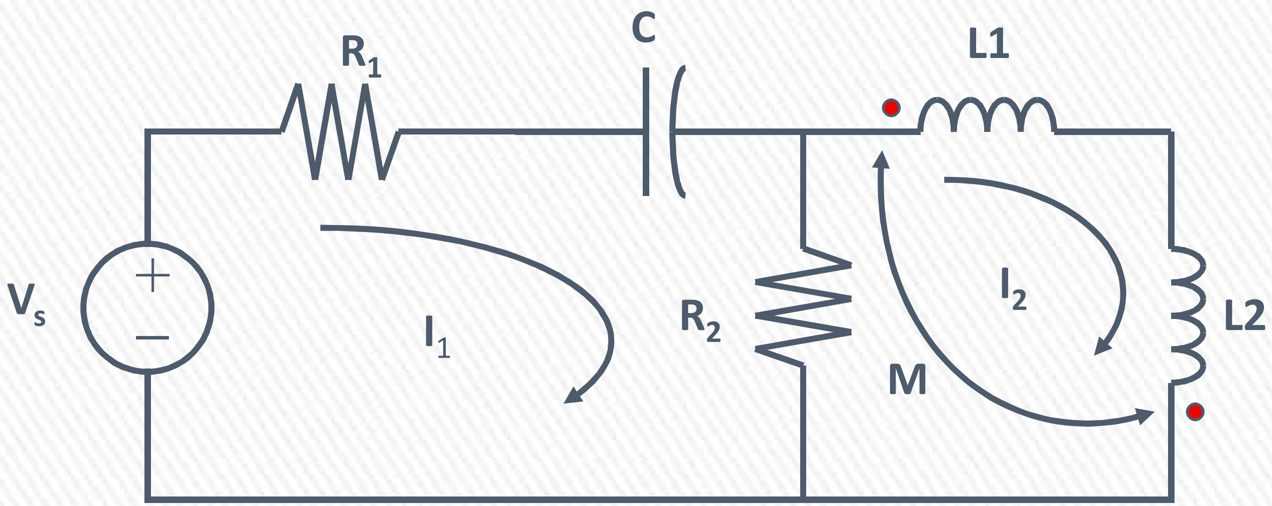



Solution:

$$\text{KVL } I_1 : (R_1 + j\omega L_1)I_1 - j\omega L_1 I_2 + j\omega M I_2 = V_s \text{.......}(1)$$

$$\text{KVL } I_2 : -j\omega L_1 I_1 + (R_2 + j\omega L_1 + j\omega L_2)I_2 - j\omega M I_2 - j\omega M(I_2 - I_1) = 0 \text{.......}(2)$$

» Circuit 4



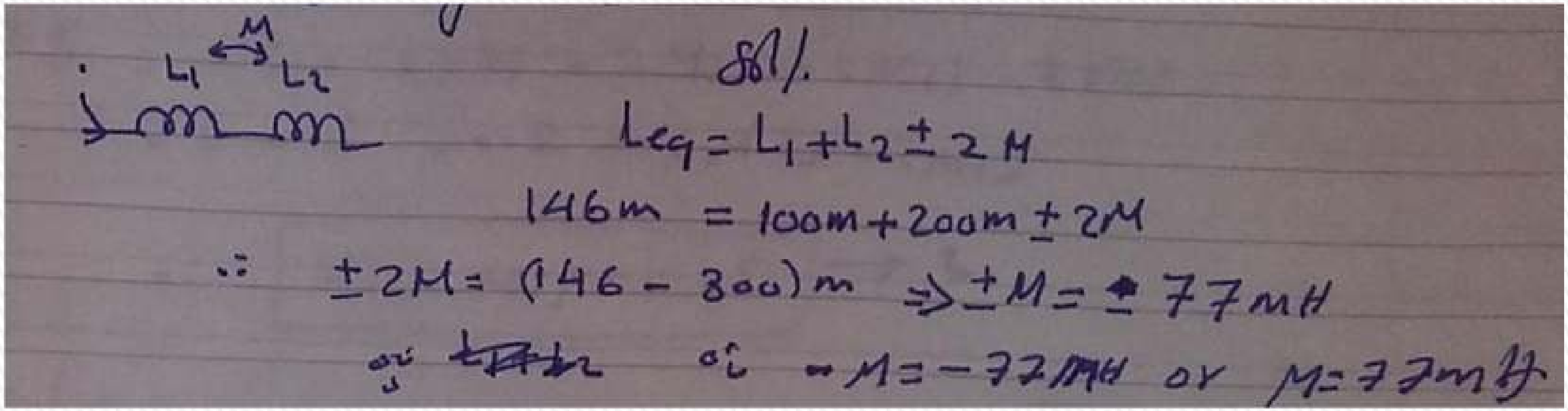

Solution:

$$\text{KVL I}_1 : (R_1 + R_2 - j/\omega c)I_1 - R_2 I_2 = V_s .......(1)$$

$$\text{KVL I}_2 : -R_2 I_1 + (R_2 + j\omega L_1 + j\omega L_2)I_2 - 2j\omega M I_2 = 0.......(2)$$

# Example

Calculate mutual inductance of two coils of self-inductance 100mH and 200mH which are connected in series to yield a total inductance of 146mH.



$L_1$ $M$ $L_2$

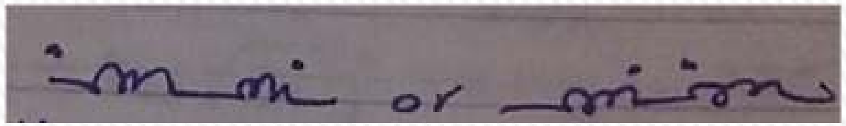
Sol.

$L_{eq} = L_1 + L_2 \pm 2M$

$146m = 100m + 200m \pm 2M$

$\therefore \pm 2M = (146 - 300)m \Rightarrow \pm M = \pm 77mH$

or $M = -77mH$ or $M = 77mH$



or

# Thank You